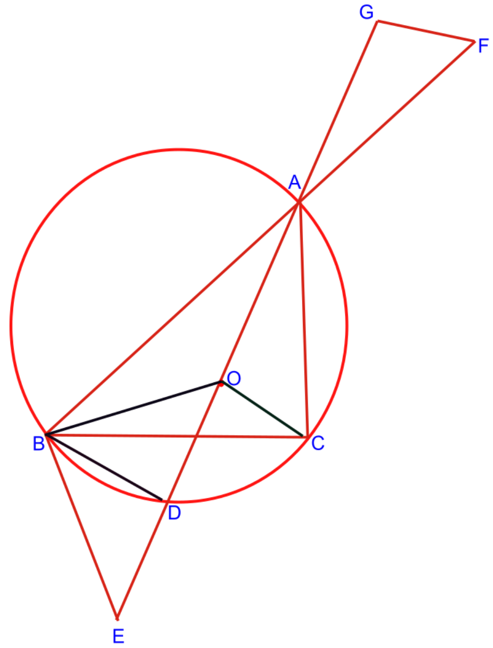
Author's Solution - Mar-2024



Given :

O is incentre

OD = DE

AC = AF

OA = AG

BEFG is concyclic.

It's enough to prove that $\angle EGF = \angle EBF$.

Construction :

Join OB, OC & BD. Consider ΔOCA & ΔAGF.

$\angle BAO = \angle OAC$ ($\because AO$ is angle bisector, as O is incentre, given)

$\angle BAO = \angle GAF$ (vertically opposite angle)

$\therefore \angle BAO = \angle OAC = \angle GAF, \ \& \ AC = AF, \ OA = AG$ Given

$\therefore \Delta OAC \cong \Delta GAF$ (By SAS Congruency)

$\Longrightarrow \angle AGF = \angle AOC = 180° - \left(\frac{A+C}{2}\right)$ [$\because OC$ is angle bisector $\angle C$

$= 180° - \left(\frac{180°-B}{2}\right)$ $\angle OAC + \angle ACO + \angle AOC = 180°$

$= 90° + \left(\frac{B}{2}\right)$ $\frac{A}{2} + \frac{C}{2} + \angle AOC = 180°$]

It's enough to prove that

$\angle EBF = 90 + \frac{B}{2}$

$\angle OBF = \frac{B}{2}$ (as CB is angle bisector of $\angle B$

$\angle DBO = \angle DBC + \angle CBO$

$= \angle DAC + \angle CBO = \frac{A}{2} + \frac{B}{2}$ --------------(1)

$\angle DOB = \angle OBA + \angle BAO = \frac{A}{2} + \frac{B}{2}$ ----------- (2) (by exterior angle property ΔAOB)

from (1) & (2) OD=DE=DB

$\Rightarrow$ D is circumcentre of $\Delta\ BEO$

$\Rightarrow \angle EBO = 90$

$\therefore \angle EBF = 90 + \frac{B}{2}$ ------------------- Hence Proved.

Solution given by

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